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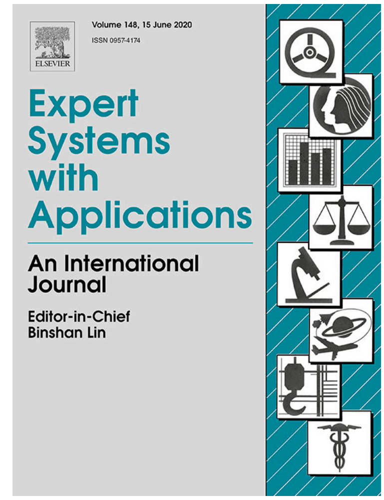

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Granular fuzzy pay-off method for real option valuation

Francisco Javier Cabrerizo[a,*], Markku Heikkilä[b], József Mezei[b,c], Juan Antonio Morente-Molinera[a], Enrique Herrera-Viedma[a], Christer Carlsson[d]

[a] *Andalusian Research Institute on Data Science and Computational Intelligence (DaSCI), University of Granada, Granada 18071, Spain*
[b] *Faculty of Social Sciences, Business and Economics, Åbo Akademi University, Turku 20500, Finland*
[c] *Avaintec Oy, Helsinki, Finland*
[d] *Institute for Advanced Management System Research, Auriga Business Centre, Turku 20100, Finland*

Abstract

In the last decade, the fuzzy pay-off method has emerged as a widely used alternative approach for real option valuation thanks to its simplicity that makes it easily approachable by practitioners from various application domains. In this study, a new direction for real option valuation is pursued by proposing a granular fuzzy pay-off method. We motivate the proposal by discussing how the extension of the original approach with granular representation can further improve the fuzzy pay-off method. The design of the granular fuzzy pay-off method is founded on the principle of justifiable granularity, which in turn relies on numeric data to build information granules that are semantically sound and experimentally justified. To illustrate the method, a case study in R&D investment in manufacturing is worked out. The extended granular fuzzy pay-off method improves performance and usability in cases with uncertainty.

Keywords: real options, fuzzy pay-off method, principle of justifiable granularity, information granules, granular computing

[*] Corresponding author: Calle Periodista Daniel Saucedo Aranda s/n, Granada 18071, Spain

Email addresses: cabrerizo@decsai.ugr.es (Francisco Javier Cabrerizo), markku.heikkila@abo.fi (Markku Heikkilä), jmezei@abo.fi (József Mezei), jamoren@decsai.ugr.es (Juan Antonio Morente-Molinera), viedma@decsai.ugr.es (Enrique Herrera-Viedma), christer.carlsson@abo.fi (Christer Carlsson)

1. Introduction

Real option can be defined as the right, but not the obligation, to undertake a business initiative: contracting, staging, abandoning, or deferring, a capital investment project, to give an example (Amram & Kulatilaka, 1999). The adjective "real" is employed as the business initiatives referenced usually involve a tangible asset rather than a financial instrument (Lütolf-Carroll & Pirnes, 2009).

The ability of analyzing, evaluating and choosing business initiatives bears a significant effect on the profitability and growth of a company. However, the precise value of a real option is difficult to estimate. Traditional option valuation methods have typically been based on discounted cash flow analysis relying on established measures such as the internal rate of return (IRR) or the net present value (NPV) to support decisions (Ho & Liao, 2011). These methods are readily applicable when project uncertainties and risks can be reduced to one single discount rate (Grinblatt & Titman, 2002). However, these traditional models require the estimation of several parameters, which can be a difficult task in an uncertain decision making environment (Carlsson & Fullér, 2003). For this reason, real option valuation has become a growing area of both practical application and academic research in recent decades. This can be attributed to the ability of real option valuation models to deal with uncertainty and offering robust estimations on the profitability of an investment and on the value of an asset even in the lack of historical data.

Real option valuation allows decision makers to respond in an optimal way when facing uncertainty by emphasizing the value of managerial flexibility. Traditional real option valuation models were built based on the notions and assumptions of financial option valuation, notably: (i) differential equations, e.g., Black-Scholes option pricing formula (Black & Scholes, 1973), (ii) lattices, e.g., the binomial option valuation method (Cox et al., 1979), and (iii) simulations, e.g., Monte Carlo methods (Boyle, 1977). However, these approaches present some drawbacks when utilized in the context of real options (Collan et al., 2009): (i) their complexity demands a good understanding of the underlying mathematics (this makes more difficult their application in practice), (ii) they do not consider procedural or structural uncertainty (Collan et al., 2016), (it must usually be of parametric type), and (iii) they were assumed to precisely simulate the underlying market as a process. Even though the assumptions of these models may hold for certain considerably efficiently traded financial securities (for instance, currencies and stocks), this assumption, in general, does not hold for real business initiatives that (i) have a market that may not at all be said to present even weak market efficiency, or (ii) simply do not have markets (Collan et al., 2009). In a nutshell, instead of focusing more on managerial relevance, traditional real option analysis utilizes complex statistical models that increase the complexity of calculus (Favato et al., 2015), which in turn hinders the development of the real option analysis field (Mathews et al., 2007).

An alternative approach present in the literature is that of using scenarios as speculative descriptions of potential future outcomes, increasing the likelihood of capturing possible threats and chances (Favato et al., 2015). The most important approaches based on scenarios for real option analysis include: (i) the Datar-Mathews (DM) approach (Mathews et al., 2007; Datar & Mathews, 2004), and (ii) the fuzzy pay-off method ($FPOM$) (Collan et al., 2009; Borges et al., 2018). Both methods differ from traditional approaches in the way they handle uncertainty: the DM approach is founded on probability theory, and the $FPOM$ is founded on fuzzy logic and fuzzy set theory (Zadeh, 1965, 2015). The DM approach uses a Monte Carlo simulation that has as input a collection of cash flow scenarios that are used to generate a pay-off distribution to model the NPV of future project outcomes (Mathews et al., 2007). Even though relying on the same basic idea of simulating scenarios, the $FPOM$ represents the expected future distribution of estimated cash flows via fuzzy numbers/possibility distributions (Collan et al., 2009).

Investment projects related to innovations typically lack historical data that could be used to build sophisticated models, and instead can be assessed only by the subjective opinion of domain experts. This setting highlights the problems related to using probability theory to handle the vagueness and imprecision inherent in the articulation of expert assessments, as in these problems the precise levels of future cash flows are unknown, that is, the uncertainty is genuine (Carlsson et al., 2007). To deal with this kind of uncertainty, the fuzzy set theory proposed by Zadeh (1975a,b,c) is a more appropriate tool as it relies on the notion of imprecision existing in human decision making and can represent the imprecise, uncertain, or vague, knowledge (as, for instance, estimated future cash flows), which human reasoning is particularly adaptive to (Zadeh, 1996; Morente-Molinera et al., 2019, 2020; Zuheros et al., 2018). Consequently, in recent years the use of fuzzy set theory has been considered as a valid alternative to deal with uncertainty in option valuation (see Carlsson & Fullér (2003); Collan et al. (2016); Borges et al. (2018); Carlsson & Fullér (2011); Kim & Lee (2018); Rodger (2013); Tolga & Kahraman (2008); Yoshida (2003); Zhang & Watada (2018)).

In this study, relying on the concept of granular structure, which links uncertainty to information in the generalized theory of uncertainty proposed by Zadeh (2005), the main objective is to design and develop a granular $FPOM$ based on the concepts of information granules and information granularity (Cabrerizo et al., 2014; Liu et al., 2018). In particular, a granular $FPOM$ that yields information granules is developed by distributing a certain information granularity level throughout the structure of the existing $FPOM$. Information granularity is considered to be a notable asset of design: the optimal distribution results in a granular $FPOM$ that, being more abstract than its numeric counterpart, is able to represent (cover) experimental data in the form of cash flow estimations. The term "granular" pertains here to a wealth of possible realizations designed via the principle of justifiable granularity (Pedrycz & Homenda, 2013; Wang et al., 2019), which offers an algorithmic and conceptual method of building information granules on the basis of numeric data. According to this principle, we design information granules by creating a sound balance between the criteria of specificity and coverage, which are generally utilized when measuring the quality of an information granule. In this manner, the constructed granular model better represents experimental data, e.g., cash flow estimations, and helps managing uncertainty.

The study is structured into five main sections. Section 2 briefly recalls the $FPOM$. Section 3 is devoted to the design and development of the granular $FPOM$, which is composed of three steps: (i) forming information granules of type-1 (in particular, intervals), (ii) extending the type-1 granules to information granules of type-2 (specifically, interval type-2 fuzzy sets), and (iii) computing the real option value from the interval type-2 fuzzy set constructed. Section 4 reports a case study in the manufacturing context to illustrate the granular $FPOM$, and Section 5 presents some conclusions.

2. The fuzzy pay-off method

To make this study self-contained, in this section we briefly describe the basics of the $FPOM$, which was proposed by Collan et al. (2009).

The $FPOM$ is based on the fuzzy set theory, which was developed to deal with imprecision and it is the main part of the theory allowing the handling of decisions in environments under uncertainty (Bellman & Zadeh, 1970). In particular, the $FPOM$ makes use of fuzzy numbers (Marín et al., 2019), which are used in fuzzy set theory to quantify subjective fuzzy observations or estimates, and fuzzy logic for creating the feasible pay-off distribution of a real option by means of three cash flow scenarios:

- A basic scenario (most likely to happen),
- an optimistic scenario (the highest possible outcome), and
- a pessimistic scenario (the lowest possible outcome).

The estimated NPV of each scenario is calculated and then used to build a triangular fuzzy number (called fuzzy NPV) illustrating the degree to which a given estimated NPV belongs to the feasible NPV collection of the real option. In the same way as the DM approach does to its pay-off distribution with the aim of including the real option flexibility within a project, the $FPOM$ maps the negative estimated NPVs of its pay-off distribution into 0. This is done to reflect that, if we expect a negative result, we have the right to not continue with the real option.

The $FPOM$ makes use of the fuzzy NPV to calculate the value of a real option, ROV, as (Carlsson & Fullér, 2011):

$$ROV = \frac{\int_0^\infty A(x)dx}{\int_{-\infty}^\infty A(x)dx} \cdot E(A_+), \tag{1}$$

where A denotes the fuzzy NPV, $\int_0^\infty A(x)dx$ calculates the area of the positive side of A, $\int_{-\infty}^\infty A(x)dx$ calculates the total area of A, and $E(A_+)$ denotes the possibilistic mean of the positive side of A, which is calculated as shown in Carlsson & Fullér (2001).

As an illustration, let us assume three cash flow scenarios provided by an expert, with each scenario as a separate cash flow statement with annual cash flows (see Table 1). The NPV of each scenario is computed as follows (Jones & Smith, 1982):

$$NPV = \sum_{t=0}^{n} \frac{cf_t}{(1+i)^t}, \tag{2}$$

Table 1: Cash flow by scenario (in millions of euros).

	Year										
Scenario	0	1	2	3	4	5	6	7	8	9	10
Optimistic	0	7	7	7	7	7	7	7	7	7	7
Basic	0	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
Pessimistic	0	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2

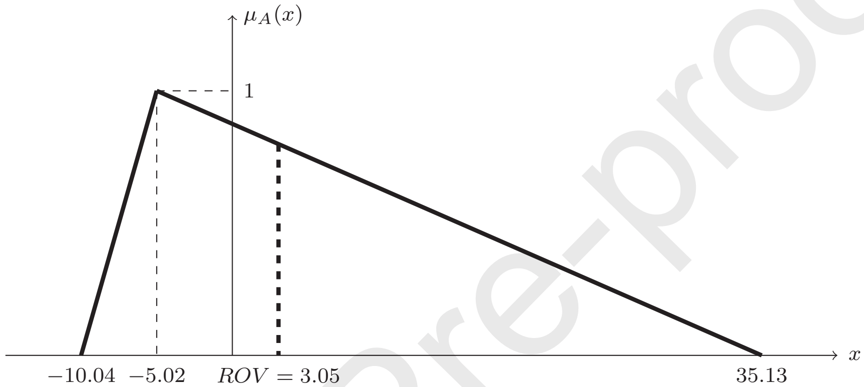

Figure 1: NPV scenarios as a triangular fuzzy number.

being cf_t the cash flow in the year t and i the discount rate or return that could be earned in alternative investments. Then, assuming a discount rate of 15%, the NPV of the optimistic scenario, NPV^O, is equal to 35.13, the NPV of the basic scenario, NPV^B, is equal to -5.02, and the NPV of the pessimistic scenario, NPV^P, is equal to -10.04. The triangular fuzzy number built by these three scenarios is represented in Fig. 1. In this example, the core, $\mu_A(x) = 1$, becomes a single value (-5.02 millions of euros) whereas the support given by the left and right tails covers the area between -10.04 and 35.13 millions of euros. The cash flow values outside the support have $\mu_A(x) = 0$, and are not considered. The real option value obtained using (1) is equal to 3.05.

The $FPOM$ has attracted great attention since it was proposed and has been employed for analysis, development and research projects (Collan & Luukka, 2014), large industrial investments (Collan, 2011), corporate acquisitions (Collan & Kinnunen, 2011), investments into information systems (Collan et al., 2014), aerospace industry (Rodger, 2013), and patents (Collan

& Kyläheiko, 2013), among others. However, it should be noted that Borges et al. (2018) proved that, because the ROV value generated by the $FPOM$ from a fuzzy NPV can in some situations be lower than the value of the possibilistic mean calculated from the same fuzzy NPV, the $FPOM$ can be interpreted to function in a finance-theoretically incorrect way. The reason is associated with the method that is employed to obtain a single value representing a fuzzy number. To avoid it, in place of using the possibilistic mean, Borges et al. (2018) employed the center of gravity to carry out this task.

3. A granular fuzzy pay-off method for real option valuation

We are concerned with elevating the $FPOM$ to a higher level of abstraction, which is referred to as the granular $FPOM$. To do so, we structure the design of the granular $FPOM$ into three steps. The first step is devoted to form a granular representative of type-1 (in particular, an interval) for each one of the three NPV scenarios. The second step is devoted to construct an information granule of type-2 (specifically, an interval type-2 fuzzy number) using the obtained information granules of type-1 in the first step. The third step is devoted to compute the real option value from the information granule of type-2 built in the second step. The next subsections describe these three steps in detail.

3.1. Obtaining a granular representative

The basic step behind the $FPOM$ is to construct the triangular fuzzy number representing the three NPV scenarios (optimistic, basic, and pessimistic) in which each scenario is a separate cash flow statement with annual cash flows. For each NPV scenario, each annual cash flow can be estimated by an expert (as in the example shown in Section 2). However, according to social psychology research, groups tend to make a better decision than the most skillful person in them (Yang, 2010). Therefore, a group of experts are usually involved in estimating the three NPV scenarios and then a representative for each scenario must be obtained.

This is formalized as follows. We assume a group of experts, $E = \{e_1, e_2, \ldots, e_m\}$. Then, each expert e_k provides an optimistic estimation, cf^o_{kt}, a basic estimation, cf^b_{kt}, and a pessimistic estimation, cf^p_{kt}, for the cash flow of every year t. Using (2), the NPV of the optimistic scenario, NPV^o_k, the NPV of the basic scenario, NPV^b_k, and the NPV of the pessimistic scenario, NPV^p_k, are obtained for each expert e_k. Then, a representative for the optimistic scenario, NPV^o, is obtained by aggregating all the NPV_k^o $(k = 1, \ldots, m)$, a representative for the basic scenario, NPV^b, is obtained by aggregating all the NPV_k^b $(k = 1, \ldots, m)$, and a representative for the pessimistic scenario, NPV^p, is obtained by aggregating all the NPV_k^p $(k = 1, \ldots, m)$. Finally, the representatives of the pessimistic, basic, and optimistic, that is, NPV^o, NPV^b, and NPV^p, form a triangular fuzzy number. The problem consists in forming an only item, which may be taken as a sound (optimal in relation to a given criterion) representative of experimental evidence that is composed of numeric data.

Data aggregation (Wang et al., 2019), from a formal perspective, is related to a mapping ϕ, which usually assumes the form $\phi : \mathbb{R}^n \rightarrow \mathbb{R}$, from an experimental evidence space (n-dimensional) to an aggregation result space (one-dimensional). Aggregation must generate some outcome representative of several sources of experimental evidence, which usually come with an intrinsic diversity, considered a block. Therefore, the result of the aggregation must be more abstract than the sources of experimental evidence. In the setting of real option valuation, it means that it is not convincing enough that a numeric value may be a good representative of a set of NPVs that are encountered in numeric form. The aggregation process can be enhanced by means of information granules (Liu et al., 2018; Cabrerizo et al., 2018; Zhu et al., 2017), which arise as a viable option worth pursuing. From a formal perspective, the aggregation is now concerned with a mapping $\phi : \mathbb{R}^n \rightarrow \mathbf{G}(\mathbb{R})$, being $\mathbf{G}(\mathbb{R})$ an information granule that can be related to any formalism of Granular Computing (Cabrerizo et al., 2020; Callejas et al., 2019; Yao et al., 2013), including rough sets, probabilities, fuzzy sets, and intervals, among others.

In the granular $FPOM$, we are interested in the aggregation of a collection of NPVs yielding a particular information granule Y, whose granularity must reflect the diversity of the NPVs to be aggregated. To quantify and accommodate the diversity of the NPVs, the information granule Y must be of a higher type than the type of the elements to be aggregated. In this case, the NPVs (numbers) are information granules of type-0. Therefore, the aggregation of the NPVs must lead to an information granule of type-1, which is a granular construct completely determined by a set of numeric parameters. Here, we are interested in information granules as intervals. Consequently, the bounds of the intervals are their numeric parameters.

The principle of justifiable granularity helps realize the granular character of Y (Pedrycz & Homenda, 2013; Wang et al., 2019). This principle is related to a way of building information granules on a basis of some available experimental data. The crux of this principle is associated with a construction of information granules so that they are conceptually meaningful (that is, they come with a well-defined semantics) and they are experimentally justifiable (that is, they are justifiable by experimental evidence). The principle of justifiable granularity involves two criteria that are in conflict: (i) specificity (Marín et al., 2018; Yager, 1992), which stresses the semantics of the information granule, and (ii) coverage (Esteve-Calvo & Lloret-Climent, 2006), which states how much data samples are embraced in the formed information granule. The definition of both criteria depend on the formal nature of the information granule to be formed (Pedrycz, 2015). Therefore, a sound compromise of these criteria must be achieved when formalizing the information granules.

Let us consider an information granule Y formalized as an interval $[a_{opt}, b_{opt}]$ and a collection of NPVs, $Z = \{z_1, z_2, \ldots, z_n\}$, $z \in \mathbb{R}$. The procedure behind the principle of justifiable granularity is structured into two steps:

1. Obtaining a numeric representative w of Z. To do so, a number of alternatives, as the mean, median, or generalized mean, could be used. In this study, the mean is used for the sake of simplicity:

$$w = \frac{1}{n} \sum_{i=1}^{n} z_i. \tag{3}$$

2. Obtaining the bounds of the interval. To do so, we individually determine the upper bound b_{opt} and the lower bound a_{opt}. First, we determine the upper bound b_{opt} by computing the specificity, sp, and coverage, cov, as:

$$sp([w, b]) = 1 - \frac{|b - w|}{|z_{max} - w|}, \tag{4}$$

and

$$cov([w, b]) = \frac{card\{z_k \mid z_k \in [w, b]\}}{card\{z_k \mid z_k > w\}}, \tag{5}$$

where $z_{max} = arg\max_{k=1,2,\ldots,n} z_k$.

The key task is to build an interval in order to get both the specificity and coverage to achieve the highest values (Fu & Lu, 2019). However, these two criteria are commonly in conflict and cannot be at maximum at the same time (Ouyang et al., 2019): increasing the coverage will reduce the specificity and vice versa. Therefore, we determine b_{opt} by maximizing the product of the specificity and coverage:

$$b_{opt} = arg \max_{b} \{sp[w,b] \cdot cov[w,b]\}. \tag{6}$$

The lower bound a_{opt} is determined similarly. Briefly, we have:

$$a_{opt} = arg \max_{a} \{sp[a,w] \cdot cov[a,w]\}, \tag{7}$$

where

$$sp([a,w]) = 1 - \frac{|w-a|}{|z_{min} - w|}, \tag{8}$$

$$cov([a,w]) = \frac{card\{z_k \mid z_k \in [a,w]\}}{card\{z_k \mid z_k < w\}}. \tag{9}$$

and $z_{min} = arg \min_{k=1,2,\ldots,n} z_k$.

The algorithms of differential evolution (Storn & Price, 1997) and particle swarm optimization (Kennedy & Eberhart, 1995) can serve here as a sound alternative to optimization.

Let $Z^b = \{z_1^b, z_2^b, \ldots, z_n^b\}$, $Z^o = \{z_1^o, z_2^o, \ldots, z_n^o\}$, and $Z^p = \{z_1^p, z_2^p, \ldots, z_n^p\}$, be the collection of NPVs for the basic scenario, the collection of NPVs for the optimistic scenario, and the collection of NPVs for the pessimistic scenario, respectively. The procedure described is applied in this step to obtain three information granules formalized as intervals $[a_{opt}^b, b_{opt}^b]$, $[a_{opt}^o, b_{opt}^o]$, and $[a_{opt}^p, b_{opt}^p]$, that reflect the diversity of the elements of Z^b, Z^o, and Z^p, respectively.

3.2. Building a type-2 information granule

This second step is devoted to constructing a type-2 information granule using the three intervals (type-1 information granules) obtained in the first step. Recall that a type-2 information granule has parameters in the form of type-1 information granules instead of numeric entities (type-0 information granules).

An early objection that was formulated related to fuzzy sets (type-1 information granules) is that it seems incongruous that something that is "fuzzy" has a very precisely defined membership function (Mendel & John, 2002). To overcome this limitation, Zadeh (1975a) introduced type-2 fuzzy sets as fuzzy

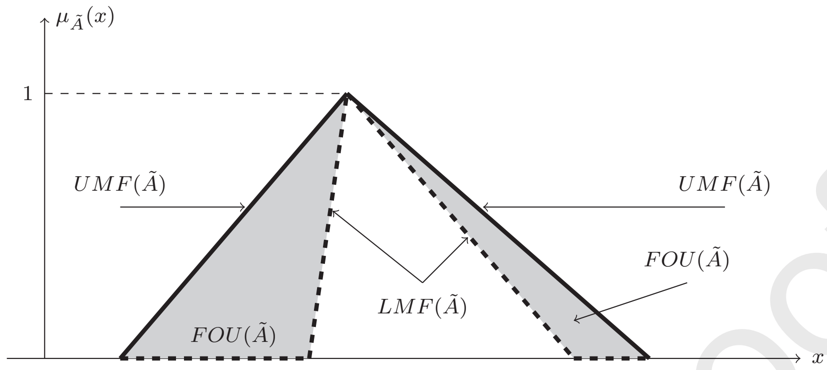

Figure 2: UMF (solid), LMF (dashed), and FOU (shaded) for an interval type-2 fuzzy set $\tilde{A}$.

sets having fuzzy membership degrees. Type-2 fuzzy sets add a third dimension to modelling imprecision providing more degrees of freedom. However, they are computationally more expensive. For this reason, interval type-2 fuzzy sets are mostly employed because the computations related to them are typically manageable even in applications requiring real time decision support (Mendel et al., 2006). Therefore, the three intervals are used to construct an interval type-2 fuzzy number.

Definition 1. *If an interval type-2 fuzzy set $\tilde{A}$ can be expressed as:*

$$\tilde{A} = (A^u, A^l) = ([a_l^u, \underline{a}^u, \bar{a}^u, a_u^u, h^u], [a_l^l, a^l, a_u^l, h^l]), \tag{10}$$

then $\tilde{A}$ is called an interval type-2 fuzzy number, where $h^u = 1$ denotes the membership value of the elements $\underline{a}^u$ and $\bar{a}^u$ in the trapezoidal upper membership function (UMF) $\mu_{A^u}(x)$, and $h^l \in [0,1]$ denotes the membership value of the element a^l in the triangular lower membership function (LMF) $\mu_{A^l}(x)$. The UMF $\mu_{A^u}(x)$ and the LMF $\mu_{A^l}(x)$ are two type-1 membership functions that bound the footprint of uncertainty (FOU) of $\tilde{A}$ (see Fig. 2), which is computed as:

$$FOU(\tilde{A}) = \bigcup_{x \in X} [\mu_{A^u}(x), \mu_{A^l}(x)]. \tag{11}$$

Then, using the three intervals obtained in the first step, the procedure to determine the interval type-2 fuzzy number $\tilde{A}$ is as follow (see Fig. 3):

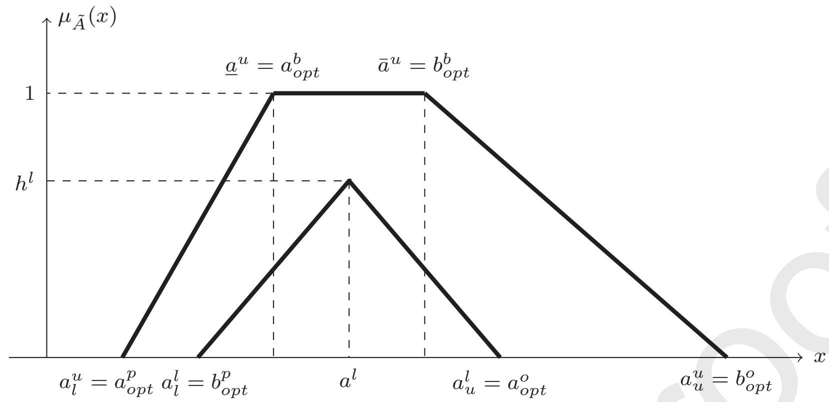

Figure 3: Interval type-2 fuzzy number $\tilde{A}$.

- A^u is determined as follows:
  - a^u_l is equal to a^p_{opt}.
  - $\underline{a}^u$ is equal to a^b_{opt}.
  - $\bar{a}^u$ is equal to b^b_{opt}.
  - a^u_u is equal to b^o_{opt}.
  - Use straight lines to connect the following points: $(a^u_l, 0)$, $(\underline{a}^u, 1)$, $(\bar{a}^u, 1)$, and $(a^u_u, 0)$.

  This results in the trapezoidal UMF $\mu_{A^u}(x)$.

- A^l is determined as follows:
  - a^l_l is equal to b^p_{opt}.
  - Obtain the intersection point (a^l, h^l) of the right leg and the left leg of the left and right most-extreme triangles (see Fig. 3) using:

$$a^l = \frac{a^l_u \cdot (\bar{a}^u - a^l_l) + a^l_l \cdot (a^l_u - \underline{a}^u)}{(\bar{a}^u - a^l_l) + (a^l_u - \underline{a}^u)}, \tag{12}$$

$$h^l = \frac{a^l_u - a^l}{a^l_u - \underline{a}^u}. \tag{13}$$

  - a^l_u is equal to a^o_{opt}.

– Use straight lines to connect the following points: $(a_l^u, 0)$, $(a_l^l, 0)$, (a^l, μ_{a^l}), $(a_u^l, 0)$, and $(a_u^u, 0)$.

This results in the triangular LMF $\mu_{A^l}(x)$.

The interval type-2 fuzzy number $\tilde{A}$ constructed using this procedure is called the interval type-2 fuzzy NPV of the project and represents its pay-off distribution.

3.3. Computing the real option value

Considering that the real option value must be computed as the weighted mean of the positive outcomes of a pay-off distribution (Datar & Mathews, 2004), the $FPOM$ computes the real option value as the possibilistic mean of the positive NPV outcomes (Collan et al., 2009). However, Borges et al. (2018) proved that there exist scenarios where the project without real options leads to a higher value than the same project with real options, which is inconsistent with the theory (Amram & Kulatilaka, 1999; Cox & Martin, 1983). This is due to the usage of the possibilistic mean to generate a crisp representation of the fuzzy number (Carlsson & Fullér, 2001). To solve this issue, Borges et al. (2018) utilized the center of gravity to obtain a crisp representation of the fuzzy number.

Taking into account these considerations, in the granular $FPOM$ the real option value is computed as:

$$ROV = \frac{area_{FOU(\tilde{A})+}}{area_{FOU(\tilde{A})}} \cdot c(\tilde{A}_+), \tag{14}$$

where $\tilde{A}$ denotes the interval type-2 fuzzy NPV, $area_{FOU(\tilde{A})+}$ symbolizes the area of the positive side of $FOU(\tilde{A})$, $area_{FOU(\tilde{A})}$ symbolizes the total area of $FOU(\tilde{A})$, and $c(\tilde{A}_+)$ denotes the centroid of the positive side of $\tilde{A}$. On the one hand, $area_{FOU(\tilde{A})+}$ and $area_{FOU(\tilde{A})}$ are calculated as follows:

$$area_{FOU(\tilde{A})+} = \int_0^{\infty} [\mu_{A^u}(x) - \mu_{A^l}(x)], \tag{15}$$

$$area_{FOU(\tilde{A})} = \int_{-\infty}^{\infty} [\mu_{A^u}(x) - \mu_{A^l}(x)]. \tag{16}$$

In order to compute $c(\tilde{A}_+)$, we use the Nie & Tan (2008)'s method as it reduces greatly the computation cost and the closed-form nature of its output enables theoretical analysis of interval type-2 fuzzy logic systems (Mendel & Liu, 2013) (refer to Nie & Tan (2008) for a detailed description of the method). However, other methods like those proposed in Mendel & Liu (2013), Karnik & Mendel (2001), and Wu & Mendel (2009), could be also used.

Similar to the $FPOM$, the granular $FPOM$ reflects the right to not continue with a business initiative if a negative result is anticipated by mapping the negative NPV values of the pay-off distribution into 0. Therefore, according to (14), if the whole interval type-2 fuzzy number $\tilde{A}$ is above 0, the real option value is equal to the centroid of the interval type-2 fuzzy number $\tilde{A}$, and if the whole interval type-2 fuzzy number $\tilde{A}$ is below 0, the real option value is equal to 0.

4. An illustrative example in the manufacturing context

A new technology is often developed to solve an existing or emerging problem in some phase of production (incrementally), such as a bottleneck in material flow, or to develop a new process to improve both output and efficiency (radically), such as a totally new process to utilize recycled material. Depending on the size and the quality of the problem area, the efforts to find solutions may require extensive tests with a pilot application. Testing phases like this intend to provide new information to managers making the decision whether to invest in the new technology. However, they may also be expensive investments by themselves. Therefore, the decision to run a pilot project often needs approval by the managers responsible for investments and research and development (R&D), including the managing director (henceforward "R&D investment management").

According to Mitchell & Hamilton (2007), R&D investment management has looked at R&D from two perspectives. From the administrative point of view, R&D is as a necessary cost of business (the cost centre approach) where R&D is an overhead expense. Specifically, such views are preferred for evaluation of early-stage or exploratory research efforts. From the viewpoint of operational management, R&D is an investment (the profit centre approach) and seeks to evaluate funds allocated to R&D with profitability criteria. Therefore, financial and market-based criteria should be used for evaluation. Addressing the profitability criteria, Mitchell & Hamilton (2007) concluded that the traditional R&D profitability analysis methods with return on investment or discounted present values (NPV and IRR) fail to deal with the implications of cutting funding from the research program and that they may also lead to failures in positioning a company in key technical areas that again may lead to ignoring the downstream options available in the research pipeline.

For the two-stage R&D utilization program, with the pilot and the operational stages, the new technology or solution is expected to provide operational benefits. Therefore, the analysis should be made with the profit center approach. The operational stage is here seen as a real option. The cost of the pilot stage, usually without any substantial cash flows, actually is the cost for acquiring this real option. Cash flows of the improved operations of the new production process are treated as incremental cash flows, that is, cash flows showing the improvement (difference) produced by the R&D investment to the previous situation without the investment, and they represent the operational benefits of the investment.

To justify this two-stage investment, the R&D investment management need to estimate operational cash flows. However, it is practically impossible to evaluate the effects of the new process precisely, as one single annual cash flow estimate. After all, the pilot project itself is to generate such new information about expected operational benefits. In addition, due to asymmetric information within the R&D investment management group about the operational benefits, and due to differing expert opinions in the group, it is in most cases impossible to reach any solution to the investment problem with market-based data, and market efficiency cannot thus be assessed explicitly. Instead, expert knowledge and a process to reach consensus (Gao & Li, 2019; Xu et al., 2019) should be used to generate the information embedded and represented in the cash flows implicitly.

In the present case, the process to reach consensus is carried out first by allowing eight managers (experts), two R&D project managers (e_1 and e_2), two R&D portfolio managers (e_3 and e_4), two factory managers (e_5 and e_6), the business controller (e_7) and the managing director (e_8), to give their cash flow estimates. Secondly, the granular $FPOM$ developed in this study helps to reach a consensus about the value of the real option.

Now, a two-stage evaluation process can be described. In the first stage, when the managers review existing data and reports they confirm the need to produce a feasibility study to find a more precise common understanding about the project. Then the managing director collects the expected (basic), optimistic and pessimistic views of all the managers about the cash flows (see Table 2).

Table 2: Cash flows provided by the managers (in millions of euros).

		Year							
Manager	Scenario	0	1	2	3	4	5	6	7
e_1	Optimistic	0	5	5	5	7	7	7	7
	Basic	0	2	2	2	4	4	4	4
	Pessimistic	0	−2	−2	−2	−1	−1	−1	−1
e_2	Optimistic	0	6	6	7	9	9	9	9
	Basic	0	3	3	4	5	5	5	5
	Pessimistic	0	−2	−2	−1	0	0	0	0
e_3	Optimistic	0	4	4	5	5	8	8	8
	Basic	0	1	1	2	2	3	3	3
	Pessimistic	0	−3	−3	−2	−2	−1	−1	−1
e_4	Optimistic	0	3	3	3	5	5	5	5
	Basic	0	0	0	0	1	1	1	1
	Pessimistic	0	−3	−3	−3	−2	−2	−2	−2
e_5	Optimistic	0	4	4	5	5	5	5	5
	Basic	0	1	1	2	2	2	2	2
	Pessimistic	0	−2	−2	−1	−1	−1	−1	−1
e_6	Optimistic	0	3	3	4	4	5	5	5
	Basic	0	−1	−1	0	0	1	1	1
	Pessimistic	0	−3	−3	−2	−2	−1	−1	−1
e_7	Optimistic	0	3	3	3	4	4	4	4
	Basic	0	−1	−1	−1	1	1	1	1
	Pessimistic	0	−3	−3	−3	−2	−2	−2	−2
e_8	Optimistic	0	4	4	5	5	6	6	6
	Basic	0	0	0	1	1	3	3	3
	Pessimistic	0	−3	−3	−2	−2	−2	−2	−2

Table 3: NPV scenarios for each manager (in millions of euros)

	Manager							
NPV	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
Optimistic	24.55	31.25	23.09	16.23	19.17	16.32	14.35	20.48
Basic	12.07	16.89	8.00	1.87	6.69	-0.32	-0.40	5.14
Pessimistic	-6.44	-3.91	-8.64	-10.60	-5.78	-8.64	-10.60	-9.94

To allow a consensus to be found, a pay-off value is then calculated with the granular $FPOM$. It is given to all participants and included in the feasibility study. In the following subsections, we describe in detail how this can be carried out.

4.1. Granular representation

First, we must compute for each manager e_k ($k = 1, \ldots, 8$) the NPV of the optimistic scenario, NPV_k^o, the NPV of the basic scenario, NPV_k^b, and the NPV of the pessimistic scenario, NPV_k^p. Using the three future cash flow scenario values provided by the managers (see Table 2) and (2), the NPVs of the three scenarios for each manager are shown in Table 3 (we have assumed a discount rate of 15%). According to it, the collection of NPVs for the basic scenario, Z^b, the collection of NPVs for the optimistic scenario, Z^o, and the collection of NPVs for the pessimistic scenario, Z^p, are:

$$
\begin{aligned}
Z^b &= \{12.07, 16.89, 8.00, 1.87, 6.69, -0.32, -0.40, 5.14\}\\
Z^o &= \{24.55, 31.25, 23.09, 16.23, 19.17, 16.32, 14.35, 20.48\}\\
Z^p &= \{-6.44, -3.91, -8.64, -10.60, -5.78, -8.64, -10.60, -9.94\}
\end{aligned}
$$

Once these three NPV collections have been obtained, a representative formalized as an interval is determined for each one of them. This is done as follows:

- Using (3), the numeric representatives w^b, w^o, and w^p, of Z^b, Z^o, and Z^p, respectively, are:

$$w^b = 6.24 \qquad w^o = 20.68 \qquad w^p = -8.07$$

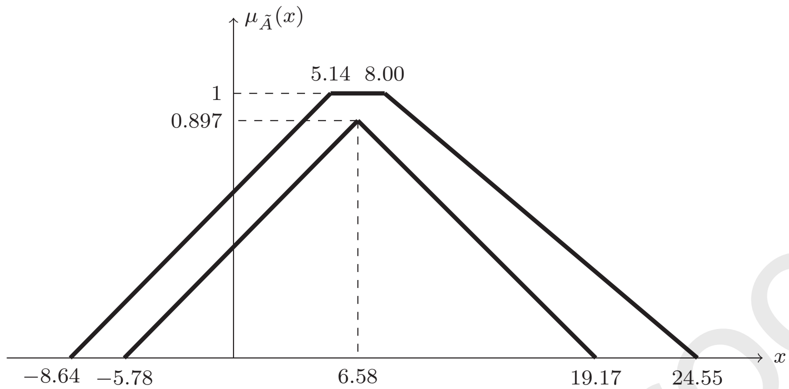

Figure 4: Interval type-2 fuzzy NPV $\tilde{A}$ associated with the example in the manufacturing context.

- Using, for example, a particle swarm optimization algorithm (Kennedy & Eberhart, 1995), we obtain the following optimal bounds of the interval, which makes an optimal balance between coverage and specificity:

$$\begin{aligned} [a^b_{opt}, b^b_{opt}] &= [5.14, 8.00] \\ [a^o_{opt}, b^o_{opt}] &= [19.17, 24.55] \\ [a^p_{opt}, b^p_{opt}] &= [-8.64, -5.78] \end{aligned}$$

4.2. Construction of the interval type-2 fuzzy NPV

Once the three intervals have been determined, the interval type-2 fuzzy NPV $\tilde{A}$ is constructed. According to the intervals obtained, the interval type-2 fuzzy NPV $\tilde{A}$ is equal to (see Fig. 4):

$$\begin{aligned} \tilde{A} = (A^u, A^l) = &([a^u_l = -8.64, \underline{a}^u = 5.14, \bar{a}^u = 8.00, a^u_u = 24.55, h^u = 1], \\ &[a^l_l = -5.78, a^l = 6.58, a^l_u = 19.17, h^l = 0.897]) \end{aligned}$$

where a^l and h^l are computed by using (12) and (13) as follows:

$$a^l = \frac{19.17 \cdot (8.00 - (-5.78)) + (-5.78) \cdot (19.17 - 5.14)}{(8.00 - (-5.78)) + (19.17 - 5.14)} = \frac{183.07}{27.81} = 6.58$$

$$h^l = \frac{19.17 - 6.58}{19.17 - 5.14} = 0.897$$

4.3. Computation of the real option value

The real option value is computed by using the interval type-2 fuzzy NPV $\tilde{A}$ obtained in the above step. According to (14), we need to compute the area of the positive side of $FOU(\tilde{A})$ and the total area of $FOU(\tilde{A})$, which is done as follows:

$$\begin{aligned} area_{FOU(\tilde{A})_+} &= \int_{0}^{5.14}\left(1-\frac{5.14-x}{8.00-(-5.78)}\right)dx+\int_{5.14}^{8.00}1dx \\ &+\int_{8.00}^{24.55}\left(1-\frac{x-8.00}{24.558.00}\right)dx \\ &-\int_{0}^{6.58}\left(1-\frac{6.58-x}{6.58-(-5.78)}\right)dx \\ &-\int_{6.58}^{19.17}\left(1-\frac{x-6.58}{19.176.58}\right)dx \\ &= 4.18+2.86+8.28-4.33-5.65=5.33 \end{aligned}$$

$$\begin{aligned} area_{FOU(\tilde{A})} &= \int_{-8.64}^{5.14}\left(1-\frac{5.14-x}{8.00-(-5.78)}\right)dx+\int_{5.14}^{8.00}1dx \\ &+\int_{8.00}^{24.55}\left(1-\frac{x-8.00}{24.558.00}\right)dx \\ &-\int_{-5.78}^{6.58}\left(1-\frac{6.58-x}{6.58-(-5.78)}\right)dx \\ &-\int_{6.58}^{19.17}\left(1-\frac{x-6.58}{19.176.58}\right)dx \\ &= 6.89+2.86+8.28-5.55-5.65=6.83 \end{aligned}$$

In addition, we must also compute the centroid associated with the positive side of the interval type-2 fuzzy NPV $\tilde{A}$. Using the Nie & Tan (2008) method, we obtain that $c(\tilde{A}_+)$ is equal to 8.66. Therefore, the real option value is:

$$ROV=\frac{5.33}{6.83}\cdot 8.66=6.76$$

In the second stage, when the feasibility study has been done, the managers gather again to share their views about the study and to update their views on the cash flows, if necessary. In such a case, that is, in the case the managers update their cash flow estimates, the granular $FPOM$ is applied again to get a new real option value. Then, the meeting is able to negotiate

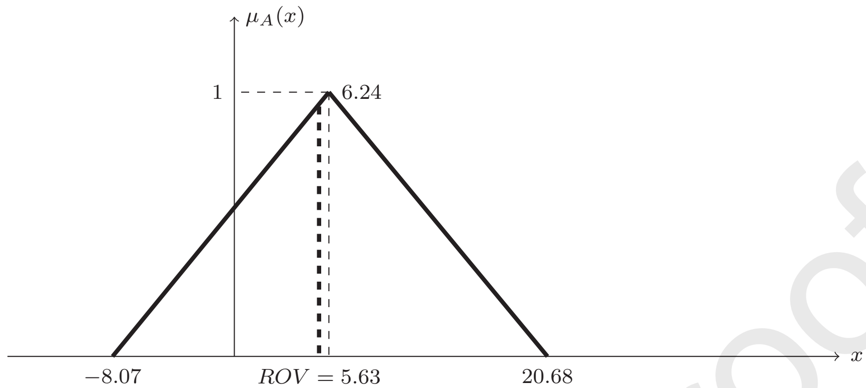

Figure 5: Fuzzy NPV A associated with this case study in the manufacturing context.

on the decision to start the pilot project, to kill the whole project or to run it partially, if possible. Yet another possibility is to keep the project alive without substantial funding and evaluate it again later if it is still relevant.

Finally, to put the obtained real option value in a certain context, we report the real option value obtained when using the original $FPOM$ proposed by Collan et al. (2009). In such a case, each NPV scenario is represented by a numeric representative (information granule of type-0), which represents to a lower extent the diversity of the cash flow scenarios given by the eight managers. Using again the mean as numeric representative of each NPV scenario, Fig. 5 depicts the fuzzy NPV A built by using the $FPOM$. Using (1), the $FPOM$ returns a value of 5.63, which is the real option value associated with this fuzzy NPV A. Comparing with the granular $FPOM$, the real option value achieves now a lower value. The granular $FPOM$, being more abstract than the original $FPOM$, is able to deal with uncertainty in a better way and can also better represent the diversity of cash flow scenarios provided by the managers, which can increase the value associated with a business case.

5. Concluding remarks

In this study, by engaging the crucial aggregation of future cash flow scenarios, we have proposed a novel approach to designing a granular $FPOM$, which offers an efficient way of building information granules as intervals in the presence of experimental data in the form of future cash flow scenarios.

The criteria of specificity and coverage have served as suitable indicators to quantify the quality of the information granules constructed. In contrast to the "standard" $FPOM$, the proposed granular $FPOM$, which is positioned at a higher level of abstraction, becomes better in dealing with the uncertainty associated with experimental data in the form of future cash flow scenarios. However, the understanding of the procedures involved in the granular $FPOM$ is not so simplistic as in those involved in the original $FPOM$ (recall that simplicity was the main reason to build the $FPOM$ (Collan et al., 2009)). Therefore, a software implementing the proposed granular $FPOM$ could be necessary when used by real-world managers.

When building the information granules, it was assumed that the experts are given the same importance when estimating future cash flow scenarios. However, it is common that, in a group of experts, they have different backgrounds and knowledge levels (Cabrerizo et al., 2013; Pérez et al., 2014). Therefore, an interesting idea would be to reformulate the building of the information granules so that it represents the knowledge level of the experts by assigning an importance weight to each estimated cash flow based on the knowledge level of the experts.

At the applied end of the spectrum of further studies, another direction in which this research could be continued in the future is to investigate the application of the granular $FPOM$ to the areas of patents (Agliardi & Agliardi, 2011; Lawryshyn et al., 2017), RFID investment (Lee & Lee, 2015), expansion strategy (Yuan, 2009), aerospace industry (Rodger, 2013) and project portfolio selection (Carlsson et al., 2007; Dou et al., 2019; Li & Yi, 2019).

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